

3 Geometry and trigonometry

Activity: Distances on Earth (Teacher version)

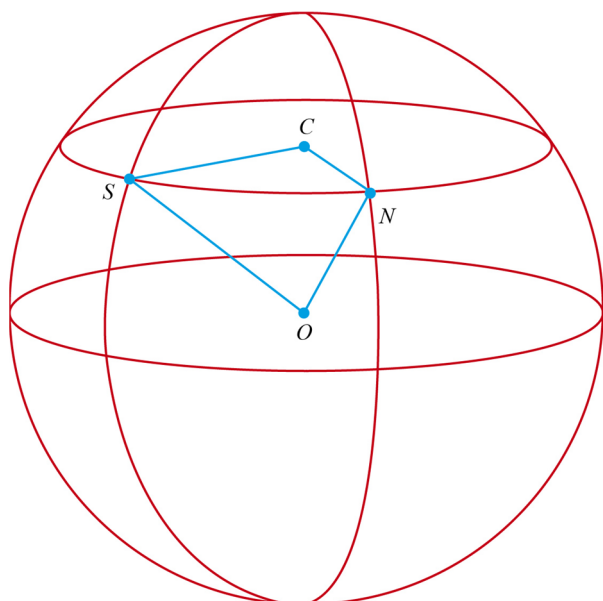
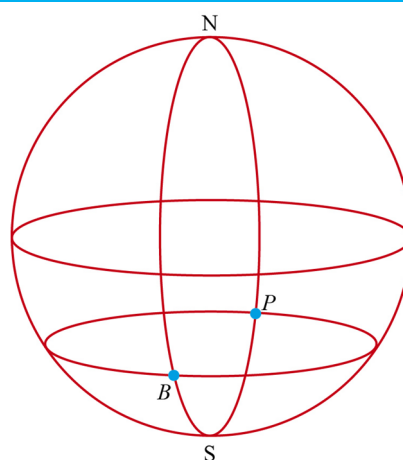
Take the radius of the Earth as 6370 km.

Questions

- 1 The diagram shows the cities of Buenos Aires (Argentina) 33°S , 60°W and Perth (Australia) 33°S , 120°E .

Find the difference between the distance from Buenos Aires to Perth:

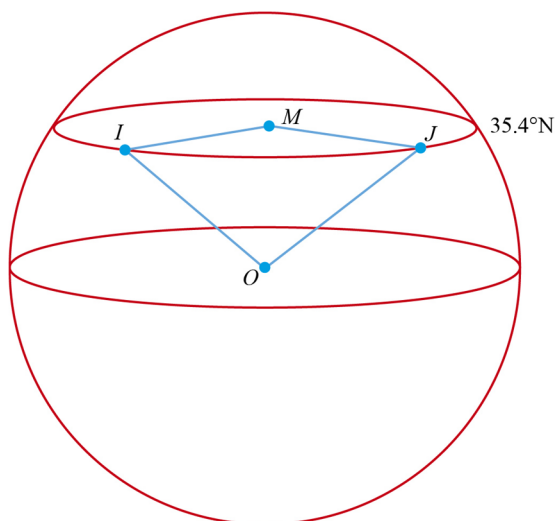
- a along the circle of latitude (*small circle*) **16 800 km**
- b over the South Pole (*great circle*) **12 700 km**



- 2 The diagram shows the cities of Norfolk (USA) 38°N , 76°W and San Francisco (USA) 38°N , 123°W . O is the centre of the Earth and C is the centre of the small circle at 38°N .
- a Find the distance across the USA along the circle of latitude from Norfolk (N) to San Francisco (S).
4120 km
 - b
 - i Use trigonometry in $\triangle CSN$ to find the chord length SN .
4000 km
 - ii Use trigonometry in $\triangle OSN$ to find the size of angle NOS .
 36.6°
 - iii Find the distance along the great circle from N to S . (Assume the shorter distance, i.e. over the North Pole.)
16 800 km

- 3 To the nearest tenth of a degree, Tokyo (Japan) is located at approximately 35.4°N , 139.5°E and Tehran (Iran) is at 35.4°N , 51.3°E .

O is the centre of the Earth, M is the centre of the small circle 35.4°N , Tehran is shown by I on the diagram and J represents Tokyo.



- a i Find angle JMI .
- ii Find the distance around the smaller of the arcs along the line of latitude 35.4°N .
- b Using triangle JIM
- i what type of triangle is JIM ?
- ii find the length of the chord IJ .
- c i Write down the lengths of OI and OJ .
- ii What type of triangle is triangle JOI ?
- iii Find angle JOI .
- iv Find the distance along the great circle from I to J .

3 a i $139.5 - 51.3 = 88.2^\circ$



Small circle radius, $r = R \cos 35.4^\circ$

Circumference of small circle

$$c = 2\pi R \cos 35.4^\circ$$

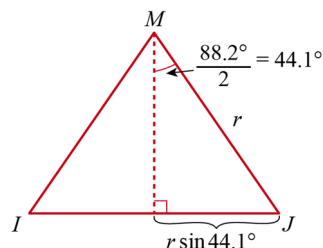
Length of overall circle arc

$$\begin{aligned} l &= \frac{88.2}{360} \times c \\ &= \frac{88.2}{360} \times 2\pi \times 6370 \cos 35.4^\circ \\ &= 7993 \text{ km} \end{aligned}$$

b i $\triangle JIM$ is isosceles.

ii $IJ = 2r \sin 44.1^\circ$

$$\begin{aligned} &= 12\,740 \cos 35.4^\circ \sin 44.1^\circ \\ &= 7227 \text{ km} \end{aligned}$$



c i $OI = OJ = R = 6370 \text{ km}$

ii $\triangle JOI$ is isosceles.

iii $\sin\left(\frac{1}{2}\angle JOI\right) = \frac{r \sin 44.1^\circ}{6370}$

$$\begin{aligned}\angle JOI &= 2 \sin^{-1} \left(\frac{6370 \cos 35.4^\circ \sin 44.1^\circ}{6370} \right) \\ &= 2 \sin^{-1} (\cos 35.4^\circ \sin 44.1^\circ) \\ &= 69.1^\circ\end{aligned}$$

iv $L = \frac{\angle IOJ}{360} \times 2\pi R$

$$\begin{aligned}&= \frac{\sin^{-1}(\cos 35.4^\circ \sin 44.1^\circ)}{180} \times 12\,740\pi \\ &= 7684 \text{ km}\end{aligned}$$

